Received: *November 27, 2000*, Accepted: *March 17, 2001*
Revised: *February 19, 2001*

On the strong coupling behavior of heterotic and type-I orbifolds*

Andrea Gregori

Dipartimento di Fisica, Università di Milano–Bicocca, and
INFN, Sezione di Milano, Italy
E-mail: agregori@pcteor.mi.infn.it

Abstract: We study the bulk effective theory of a class of orbifolds of the type-IIB string with D5-branes, compactified to four dimensions. These constructions are connected, in a region of their moduli space, to some orbifolds of the type-I and heterotic string. We compare the effective actions through the coupling of the R^2 term, and we argue that these orbifolds provide non-perturbative deformations of the latter, in which the gauge group is entirely non perturbative.

Keywords: Superstrings and Heterotic Strings, String Duality, Supersymmetric Effective Theories.

*Research partially supported by the EEC under the contract TMR-ERBFMRX-CT96-0045.

JHEP03(2001)024

Contents

1. Introduction

In ref. [1] we analyzed a class of orbifolds of the type-IIB string, in the presence of parallel D5-branes, with partial, spontaneous breaking of the supersymmetry to $\mathcal{N}_4 = 2$. We discussed how, in a region of the moduli space, they are connected to the type-I and heterotic string, arguing that these constructions provide a possible non-perturbative deformation of type-I and heterotic orbifold constructions. These models were obtained by considering freely acting orbifolds of the type-IIB string projected with $I_{(7,8,9,10)}\Omega$, the world-sheet parity times the reflection along four compact coordinates, and then looking at the theory from the bulk point of view. In the compactification limit of vanishing size of the coordinates x_7, x_8, x_9, x_{10}, these constructions could be matched with the ordinary type-I orientifold construction, obtained by projecting the type-IIB string with $\tilde{\Omega} \equiv I_{(7,8,9,10)}\Omega$ (in the following, with an abuse of language, we will refer to these models as to the $\tilde{\Omega}$ constructions). Owing to the free action of the orbifold projection, at a generic point in the moduli space the corresponding ordinary, type-I construction did not have a D5-branes sector, appearing only in a corner of the moduli space. In the $\tilde{\Omega}$ construction, where D9- and D5- branes appear exchanged, it was the D9-branes sector to be missing at a generic point in the moduli space: the open string sector was therefore entirely provided by strings ending on D5-branes. By choosing a configuration of branes and orientifold planes such as to provide a local cancellation of the tadpoles, it was possible, thanks to the existence of directions transverse to the branes, to look at the bulk theory in the limit of large volume of the transverse space, $V_{(4)} \to \infty$. As discussed in ref. [1], assuming the bulk point of view amounts to performing a translation resulting into an half-integer shift of the winding numbers corresponding to the transverse coordinates: in the $V_{(4)} \to \infty$ limit the action of the D5-branes and of the associated O5 planes is "trivialized". This construction is therefore somehow reminiscent of the construction of type-IIB strings with D3-branes considered in ref. [2]. Here however the space transverse to the branes is compact. From the bulk point of view, the breaking of supersymmetry due to the $\tilde{\Omega}$ projection is seen as "spontaneous", corresponding, in the effective theory, to a non-perturbative super-Higgs mechanism. The decompactification limit exists only for a configuration of the D5-branes and orientifold planes such that there is a local cancellation of the Ramond-Ramond charge. In the case of D5-branes, the gauge group must be broken to $\mathrm{U}(1)^{16}$. More generically, a decompactification limit may exist also for larger gauge groups, when they correspond to Dp-branes configurations, p < 9, with a local cancellation of the RR charge with the appropriate orientifold planes. For SO(32), this limit does not exist [3].

In this work, we continue the analysis of ref. [1], by considering the case of non-freely acting orbifolds. The breaking of supersymmetry is in this case non spontaneous, and it is not possible to restore a larger amount of supersymmetry by going to special limits in the moduli space. On the other hand, these orbifolds have fixed points, that provide $\mathcal{N}_4 = 2$ vector and hypermultiplets. The examples we consider actually include a construction that matches, in a corner of the moduli space, a phase of the type-I "$\mathrm{U}(16) \times \mathrm{U}(16)$", $\mathcal{N}_4 = 2$ orbifold [4, 5], in which appropriate Wilson lines break the gauge group to $\mathrm{U}(1)^{16} \times \mathrm{U}(1)^{16}$. From the heterotic point of view, the extension of the gauge group is non perturbative.

By comparing the expression of the coupling of the R^2 term in the four-dimensional effective actions of the $\tilde{\Omega}$ constructions, considered in different limits of the moduli space, with those of the related heterotic and type-I theories, we conclude that these models can be interpreted as "semi-freely acting orbifolds" of possible underlying higher-dimensional theories, in which the non-spontaneous breaking of supersymmetry is due to the interference of a "freely acting" projection, corresponding to the "$\tilde{\Omega}$" construction, and a non-freely acting one, the $\mathbb{Z}_2$ orbifold projection. The different nature of these two projections is reflected in the asymmetry between the D9- and D5-branes sectors. At a generic point in the moduli space, the "D9" sector is massive, and the only gauge group is provided by the D5 branes, that, in the $\tilde{\Omega}$ orbifolds, appear as space-fulfilling, D9 branes.

The paper is organized as follows: in section 2 we discuss the $\tilde{\Omega}$ constructions. In section 3 we describe their type-I duals, while the general properties of the heterotic duals are discussed in section 4. In section 5 we compare these theories through the analysis of the corrections to the R^2 term. As in ref. [1], we compare here the heterotic and type-I computations with the result obtained on the "$\tilde{\Omega}$" side in the two limits of small and large volume, $V_{(4)}$, of the space transverse to the D5-branes. For small $V_{(4)}$, the construction matches the ordinary, type-I orientifolds, while in the opposite limit, there is a non-trivial behavior, due to the suppression of the D5-branes effects.

We devote section 6 to further comments and conclusions.

2. The $\tilde{\Omega}$ orientifolds

We start by reviewing the action of the projection $\tilde{\Omega}$ on the type-IIB string. We consider directly the situation that will be of interest for us in the following, namely the case in which six coordinates are compactified on a product of circles. The ordinary orientifold construction is obtained by modding the type-IIB string by the world-sheet parity, Ω. In less than ten dimensions, it is possible to consider, instead of Ω, the operation $\tilde{\Omega}$, defined as the world-sheet parity times the target space reflection, $I_x :\ x \to -x$, along an even number of compact coordinates x. When we mod the type-IIB string with $\tilde{\Omega} \equiv \Omega \times I_{(7,8,9,10)}$, we obtain an orientifold in which the O9 plane is replaced by O5 planes and the D9-branes by D5-branes. It is still possible to have a local tadpole cancellation, simply by putting each one of the sixteen D5-branes, together with its mirror, on one of the sixteen O5 orientifold planes. In this configuration, the initial $\mathcal{N}_4 = 8$ supersymmetry of the type-IIB string is reduced to $\mathcal{N}_4 = 4$, and the massless spectrum is the same as that of the ordinary type-I string in four dimensions, with the gauge group broken to $\mathrm{U}(1)^{16}$. The partition function of this orientifold corresponds to that of the type-I string, with four coordinates T-dualized: their contribution appears therefore with the inverse of the radius and with winding numbers instead of momenta. As we remarked in ref. [1], owing to the breaking of T-duality along the coordinates transverse to the D5-branes, in the type-IIB string with $\tilde{\Omega}$ it is possible to assume a point of view from which the partial breaking of supersymmetry appears as "spontaneous". This corresponds to looking at the system from the "bulk". The partition function of this "displaced" theory is easily obtained from the ordinary orientifold partition function by performing a "translation" toward the center of the segments in the coordinates transverse to the D(5)-branes. This operation amounts to a shift in the windings in the Klein bottle and in the open string diagrams. In the "transverse channel" this appears instead as a rescaling of the momenta by an amount proportional to the fraction of the segment it has been translated. In the closed string sector, the Torus, being by itself invariant under translations, is not affected by this operation. From the point of view of this picture, the theory coincides with the ordinary type-I orientifold in the $V_{(4)} \to \infty$ limit. Indeed, in this limit the most correct description is obtained when the transverse coordinates are T-dualized, giving the ordinary type-I construction.

JHEP03(2001)024

In the $V_{(4)} \to \infty$ limit there is instead an approximate restoration of the initial supersymmetry, with the disappearance of the D5-branes sector. This phenomenon cannot be observed in the ten-dimensional constructions, because the orientifold projection produces there D9-branes, that fulfill the space: there are therefore no transverse directions along which to move and go "far from the branes". This motion can be observed only when, after compactification, appropriate Wilson lines are switched on: these can be interpreted as the un-freezing of "transverse" directions.

When we add a further $\mathbb{Z}_2$ orbifold projection, acting as a twist along x_7, x_8, x_9, x_{10}, supersymmetry is further broken to $\mathcal{N}_4 = 2$. In ref. [1] we considered the case in which the twist is coupled to a translation along x_4 and/or x_5, inside the untwisted two-torus. The associated breaking of supersymmetry is in that case spontaneous, and, from the bulk point of view, the theory appears to possess a spontaneously broken $\mathcal{N}_4 = 8$ supersymmetry. What we consider now is instead the case in which there is no translation associated to the twist. This $\mathbb{Z}_2 \equiv \mathbb{Z}_2^o$ orbifold possesses then sixteen fixed points. The combined action of the $\mathbb{Z}_2^o$ and $\tilde{\Omega}$ projections generates also a D9-branes sector, absent in the previous case. This sector provides, at the abelian point, sixteen vector multiplets. The analysis of the decompactification limit in this $\mathcal{N}_4 = 2$ theory is not as simple as it was for the $\mathcal{N}_4 = 4$ case: strictly speaking, owing to the simultaneous presence of both D5- and D9-branes, this limit is always singular. Even though in the $V_{(4)} \to \infty$ limit the D5-branes sector is "trivialized", in the sense above described for the $\mathcal{N}_4 = 4$ case, the breaking of supersymmetry is in this case non spontaneous: as we will see, there is no limit in the moduli space in which the $\mathcal{N}_4 = 2$, D9-branes sector, disappears, and the theory is therefore always in the phase of reduced, $\mathcal{N}_4 = 2$ supersymmetry.

Besides the simple case with a maximal number of D9-branes, we will consider also constructions in which their number is reduced by projections acting on the number of orbifold fixed points. The $S^1/\mathbb{Z}_2$ orbifolds possess in fact a symmetry that exchanges the states in the twisted sector, constructed on the twist fields σ_+, σ_-, and projects the momenta in the states V_{mn} of the untwisted sector (see ref. [11]):

$$\mathcal{D}: \; (\sigma_+, \sigma_-, V_{mn}) \longrightarrow (\sigma_-, \sigma_+, (-1)^m \, V_{mn}) \, . \tag{2.1}$$

In our $\mathbb{Z}_2^o$ orbifold there are four directions possessing this symmetry: by coupling the $\mathcal{D}$-operation on the twisted coordinates to a translation in the two-torus, it is therefore possible to construct orbifolds with a reduced number of twisted states [12]–[14]. This operation commutes with the supersymmetry-breaking projection. By projecting with one such operation,[1] the number of fixed points is reduced by half; with two of them it is reduced to a quarter of the initial one. This is the maximal reduction, because there are only two independent directions in the two-torus, along which it is possible to act with the associated translation. Starting from the model

[1]We refer the reader to [12]–[14] for a detailed discussion of this operation.

discussed above, with sixteen hypermultiplets originating from the twisted sector, and a corresponding D9-branes sector with sixteen vector multiplets, it is possible to construct, via $\mathcal{D}$-projections, models with respectively eight and four of them. All these models differ only in the $\mathbb{Z}_2$-twisted and the D9-branes sector, the rest of the spectrum being the same (see also section 3).

3. The type-I duals

The type-I dual of the orbifold with sixteen fixed points is the model with maximal gauge group $\mathrm{U}(16) \times \mathrm{U}(16)$, presented in [4, 5], which is constructed as an orientifold of the type-IIB string compactified on $\mathbb{T}^2 \times \mathbb{T}^4/\mathbb{Z}_2$. The second $\mathrm{U}(16)$ factor, that originates from the D5-branes sector, corresponds, on the heterotic side, to the contribution of small instantons [15], and is entirely non perturbative. The gauge group $\mathrm{U}(16) \times \mathrm{U}(16)$ is obtained when all the D5-branes sit at an orbifold fixed point [5]. The open string massless spectrum contains vector multiplets in the adjoint representation of $\mathrm{U}(16) \times \mathrm{U}(16)$ and hypermultiplets in the $(\mathbf{120}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{120})$, antisymmetric representations, and in the $(\mathbf{16}, \mathbf{16})$, bifundamental representation. The massless spectrum of this model includes then the states of the closed string sector, namely the sixteen hypermultiplets originating from the orbifold twisted sector and the usual three vector and four hypermultiplets of the compactification. By introducing appropriate Wilson lines and separating the D5-branes from each other, it is possible to break the gauge group $\mathrm{U}(16) \times \mathrm{U}(16)$ to $\mathrm{U}(1)^{16} \times \mathrm{U}(1)^{16}$. At this point, the second $\mathrm{U}(1)^{16}$ factor can be shown to be anomalous [6]. The anomaly cancellation mechanism gives also a mass to the hypermultiplets originating from the twisted sector. Due to the presence of the Wilson lines breaking to the abelian subgroup the first factor, only sixteen hypermultiplets remain.

The type-I duals of the orbifolds with reduced number of fixed points are easily constructed as orientifolds of the type-IIB string, compactified on $\mathbb{T}^2 \times \mathbb{T}^4/\mathbb{Z}_2$ and projected with $\mathcal{D}$-operations. Choosing $(-1)^m$ for the action on $\mathbb{T}^2$ associated to the $\mathcal{D}$-projection leads to a shift that lifts the mass of half of the states of the closed string twisted sector by projecting their momenta. This operation affects therefore the torus amplitude and the D5-branes contribution in the Annulus and Möbius strip amplitudes. The partition function of this model is given by half the partition function of the ordinary orbifold plus half the partition function of the "momentum breaking" freely acting orbifold of ref. [16]. The tadpole cancellation conditions for the D9-branes gauge group are the same; therefore, the part that corresponds to the perturbative heterotic gauge group is not affected. The tadpole conditions for the D5-branes instead are changed: now they are solved by putting half of the usual number of D5-branes. The rank of the non-perturbative gauge group is reduced by half per each such $\mathcal{D}$-projection. At the abelian point the gauge group is therefore $\mathrm{U}(1)_9^{16} \times \mathrm{U}(1)_5^{16/2^{n_{\mathcal{D}}}}$, with $n_{\mathcal{D}}$ the number of $\mathcal{D}$-projections.

JHEP03(2001)024

4. The heterotic duals

We pass now to consider the heterotic constructions, of which the effective theories considered in section 2 are expected to constitute possible non-perturbative deformations. The connection between these theories and the heterotic string is realized through their orientifold limit. The heterotic duals are therefore dual to the type-I orientifolds of the previous section. The heterotic dual of the $\mathrm{U}(16) \times \mathrm{U}(16)$ type-I orientifold, with gauge group broken to $\mathrm{SU}(16)$, was explicitly constructed in ref. [9], by starting from the $\mathcal{N}_4 = 4$ heterotic string with gauge group $\mathrm{SO}(32)$. As discussed in ref. [6], this construction is in the same moduli space of the $\mathcal{N}_4 = 2$ heterotic theory derived from the $\mathcal{N}_4 = 4$ theory with gauge group $E_8 \times E_8$, with an embedding of the spin connection into the gauge group realized by 12 instantons in each of two E_8's [10]. This second construction can be realized as a $\mathbb{T}^2 \times \mathbb{T}^4/\mathbb{Z}_2$ orbifold of the $\mathcal{N}_4 = 4$ heterotic string compactified on $\mathbb{T}^6$ [17]. This is the construction we will consider here. From this, we will derive also the duals of the orbifolds with a reduced number of fixed points.

We start therefore with the dual of the orbifold with sixteen hypermultiplets in the twisted sector: it corresponds to the maximal rank heterotic construction, in which the gauge group embedding of the spin connection is realized as follows. The $\mathbb{Z}_2$ orbifold projection acts symmetrically on the left and right movers of $\mathbb{T}^4$ and has a $c = (0, 6)$ embedding in the currents. This means that if the $c = (0, 16)$ block corresponding to the currents is described in terms of 32 free fermion degrees of freedom, the $\mathbb{Z}_2$ projection acts as -1 on 12 of these fermions. An embedding of this form is not necessarily required by modular invariance, that would be satisfied also by a $c = (0, 2)$ embedding: our choice is dictated by the fact that we want the $\mathbb{Z}_2$-twisted sector to be massless. However, this is not enough. If we count the number of hypermultiplets of the $\mathbb{Z}_2$-twisted sector, we see that they exceed the number of orbifold fixed points. This is due to the fact that, owing to the embedding of the spin connection into the gauge group, on the heterotic side these hypermultiplets are charged under the gauge group. In order to reproduce the "type-I" duals, we must break to $\mathrm{U}(1)$'s the factor of the gauge group under which these hypermultiplets are charged. This is realized through the introduction of further Wilson lines. Now, if we count the number of such hypermultiplets at the $\mathrm{U}(1)$ point, we see that they are half of those of the type I. A way of seeing this is by counting the scalars of these hypermultiplets: we have a factor sixteen coming from the sixteen fixed points of the $\mathbb{T}^4/\mathbb{Z}_2$ orbifold, times a factor two, the multiplicity of a spin(4) corresponding to the four twisted left-moving fermions, of which the GSO projection forces to choose one of the two chiralities. Since the gauge group is $\mathrm{U}(1)^{16}$, there are no additional multiplicities coming from the part embedded in the gauge group, as we wanted. We have therefore half of the scalars of sixteen hypermultiplets. In order to obtain the correct number, we must introduce a further, discrete Wilson line Y, whose action can be easily described in terms of free fermions: it is introduced by a set of four complex fermions, two of them intersecting the $\mathbb{Z}_2$ orbifold projection on the currents. Besides this, Y acts also as a shift in one of the twisted coordinates of the compact space. To be concrete, let's start by the case in which, at the $\mathcal{N}_4 = 4$ level, the gauge group of the currents is broken to $\mathrm{SO}(16) \times \mathrm{SO}(16)$. The $\mathbb{Z}_2$ orbifold projection is then embedded in an SO(8) subgroup of the first SO(16) and an SO(4) subgroup of the second SO(16). The Wilson line Y picks then a factor SO(4) in each SO(16): in the first SO(16) factor the SO(4) embedding of Y is a subgroup of the SO(8) of the orbifold projection. The gauge group is in this case broken to $[\mathrm{SO}(8) \times (\mathrm{SO}(4)_Y \times \mathrm{SO}(4))_{\mathbb{Z}_2}] \times [\mathrm{SO}(8) \times \mathrm{SO}(4)_Y \times \mathrm{SO}(4)_{\mathbb{Z}_2}]$: Y plays therefore the role of symmetrizing the $\mathbb{Z}_2$ orbifold projection in the two SO(16) factors. Besides the $\mathbb{Z}_2$-twisted sector, there is now also the massless $\mathbb{Z}_2 \times Y$-twisted sector, that provides the other half of the states we needed in order to form the sixteen hypermultiplets of the twisted sector.

As in the dual orbifolds, the number of hypermultiplets originating from the $\mathbb{Z}_2$-twisted sector can be reduced by half or to a quarter of the initial one by modding out the twisted states with $\mathcal{D}$-projections. Such projections act on $\mathbb{T}^2$ as translations.

In all these models, at the $\mathrm{U}(1)^{16}$ point, the spectrum of the massless states originating from the $c = (0, 16)$ "currents" contains only sixteen vector multiplets. In order to break completely the gauge group, it is in fact necessary to introduce Wilson lines that eliminate all the hypermultiplets: the only remaining are those of the compact space (four), and those of the orbifold twisted sector, whose number coincides with the number of fixed points.

5. Comparison of the models and the effective strong coupling behavior

Up to now we have simply shown that there exists a corner in the moduli space in which the massless spectra of these constructions coincide. In the following we want to discuss the map between the moduli in the vector multiplets. Before considering the $\tilde{\Omega}$ constructions, we review the comparison of heterotic and type-I strings, already analyzed in previous works [8, 9]. We first recall some basic aspects of this duality, which can be verified by looking at the renormalization of what in refs. [12]–[18] was called the "regular" gravitational term, a combination of R^2 and F^2 amplitudes, that possesses the property of being an "holomorphic", smooth function of the moduli in all the string constructions (see below). We then consider the analogous corrections in the $\tilde{\Omega}$ constructions, both in the limit $V_{(4)} \to 0$ and $V_{(4)} \to \infty$. In the first limit, where the $\tilde{\Omega}$ theory matches the type-I string, the correction coincides with that of the latter. In the $V_{(4)} \to \infty$ limit, the D5-branes effects are instead suppressed, and the model behaves as an $\mathcal{N}_4 = 2$, type-I orbifold without D5-branes, like the

JHEP03(2001)024

Scherk-Schwarz breaking model considered in refs. [16, 18]. From the point of view of the effective four-dimensional theory that matches the heterotic and type-I string in a corner of the moduli space, this is a non-perturbative limit.

The R^2 corrections. As we said, in order to compare different string constructions, we must look at an amplitude regular in all the theories. As it was discussed in refs. [1, 12, 18], the correct amplitude is obtained by projecting out of the R^2 amplitude the terms that mix "bulk" and "branes" contribution. This subtraction corresponds, on the heterotic string side, to the term:

$$\langle R^2 \rangle_{(\mathrm{o})} = \langle R^2 \rangle + \frac{1}{12} \langle P^2 \rangle_{(\mathbb{T}^2)} + \frac{5}{48} \langle F^2 \rangle_{\mathrm{gauge}} . \tag{5.1}$$

The second term in the r.h.s. corresponds to the amplitude of the $\mathrm{U}(1)^2$ of the two-torus, computed as in [12]; the third is proportional to the gauge amplitude of the currents. The subtraction of the torus amplitude is also necessary, because it has singularities absent in the dual string constructions. On the type-I side, the one-loop contribution to the R^2 term is proportional to an index [19]: this subtraction amounts therefore to a simple rescaling of the beta-function coefficient. In order to compare with the heterotic side, this rescaling may be necessary, because we must subtract the contribution of the D5-branes states, non perturbative on the heterotic side. In the present case (at the abelian point), all the extra gauge multiplets are anomalous and there is no such a non-perturbative extension. On the $\tilde{\Omega}$ duals, the same considerations are valid, with the role of D9- and D5-branes exchanged. As we learned in [1], in the $V_{(4)} \to \infty$ limit, the D5-branes effects are suppressed and we expect the effective coupling of this amplitude to depend only on U, the modulus associated to the complex structure of the two-torus around which the D5-branes are wrapped, and on a "tree level" contribution, proportional to the vacuum expectation value of a field, $\mathrm{Im}\, \tilde{S}'$, corresponding to the coupling of the D9-branes sector, always present.

5.1 The type-I amplitudes

Owing to extended supersymmetry, the corrections to the R^2 and F^2 amplitudes receive a contribution only from the tree and one loop levels. Putting together tree level and one-loop contributions, we get the following expression for the effective coupling:

$$\frac{16\,\pi^2}{g^2} = 16\,\pi^2\, \mathrm{Im}\, S + 16\,\pi^2\, \mathrm{Im}\, S' + \Delta^{N^{(\mathrm{t})}}(U) + \frac{b \log M_{\mathrm{P}}}{\sqrt{p^2}} . \tag{5.2}$$

The first two terms on the r.h.s. give the tree-level contribution. The fields S and S', whose imaginary parts parametrize the gauge coupling in the D9-branes and the D5-branes sector, respectively, are defined as in refs. [7]–[9]:

$$\begin{aligned} \mathrm{Im}\, S &= \mathrm{e}^{-\phi_4} G^{1/4} \omega^2 , \\ \mathrm{Im}\, S' &= \mathrm{e}^{-\phi_4} G^{1/4} \omega^{-2} , \end{aligned} \tag{5.3}$$

where ϕ_4 is the type-I dilaton of four dimensions, $\sqrt{G} = R_4 R_5$ the volume of the untwisted two-torus and ω^4 the volume of the K3 ($\sim \mathbb{T}^4/\mathbb{Z}_2$). The real parts, $\mathrm{Re}\, S$, $\mathrm{Re}\, S'$, are the scalars dual, respectively to $B_{\mu\nu}$ and B_{45}. The function $\Delta^{N^{(\mathrm{t})}}(U)$ is, for $N^{(\mathrm{t})} = 16$ orbifold fixed points:

$$\Delta^{16}(U) = -2 \log \mathrm{Im}\, U |\eta(U)|^4 \,. \tag{5.4}$$

In the other cases, it coincides with the $\Delta^{N^{(\mathrm{t})}}(U)$ term of the heterotic corrections given in eqs. (5.16) and (5.17) below. The beta-function coefficient b, that depends on $N^{(\mathrm{t})}$, is the same as in eqs. (5.5) and (5.13).

5.2 The "$\tilde{\Omega}$" amplitude in the limit $V_{(4)} \to \infty$

According to the observations of ref. [1], in the large-$V_{(4)}$ limit, the type-IIB orbifold with $\tilde{\Omega}$ orientifold projection, as seen from the bulk point of view, behaves as an $\mathcal{N}_4 = 2$, type-I orientifold without D9-branes. Therefore, in the this limit, we expect the correction to depend only on a modulus parametrizing the coupling constant of the $\tilde{\Omega}$ D9-branes sector and on the modulus U, given as before. We have then:

$$\frac{16\,\pi^2}{g^2} \approx 16\,\pi^2\, \mathrm{Im}\, \tilde{S} \,+\, \Delta^{N^{(\mathrm{t})}=16}(U) + \frac{b \log M_{\mathrm{P}}}{\sqrt{p^2}}\,, \tag{5.5}$$

where the function $\Delta^{N^{(\mathrm{t})}}(U)$ is given as in eq. (5.2) and $\approx$ indicates that in eq. (5.5) we are neglecting terms suppressed with the volume of the transverse space, $V_{(4)}$.

Owing to the symmetry between the D9- and D5-branes sectors of the type-I construction, we could think to repeat the argument and perform a translation toward the bulk in the type-I construction, in order to observe the disappearence of the D5-branes sector. This is however not possible: this sector is produced by an orbifold projection that we expect to preserve S-duality of its coupling. More precisely, even though the branes in the open string sector are pushed at infinity, in the closed string sector the orbifold projection still works, breaking supersymmetry to $\mathcal{N}_4 = 2$ at any point in the bulk, and the branes states we have apparently lost reappear in the orbifold twisted sector. For what matters the effective coupling of the R^2 term, we may argue that the above property reflects into the invariance of the effective coupling under the inversion of the modulus parametrizing the effective coupling of the D9-branes sector. This is in fact what we will find by matching, in a region of the moduli space, this construction with the heterotic string. The first term in eq. (5.5) can in fact be seen as the $T \to \infty$ limit of:

$$\Delta^{N^{(\mathrm{t})}=16}(T) = -2 \log \mathrm{Im}\, T |\eta(T)|^4 \,, \tag{5.6}$$

with the identification $T \sim \tilde{S}^{-1} \equiv S'$. The last term in the r.h.s. of eq. (5.5) gives the infrared running in terms of the Planck mass M_{P} and a physical cut-off, $\sqrt{p^2}$; the coefficient is $b = 6 + N^{(\mathrm{t})}/4$, where $N^{(\mathrm{t})}$ is the number of the orbifold fixed points.

5.3 The heterotic amplitude

On the heterotic side the computation is more involved. Even though the moduli in the hypermultiplets do not appear in the renormalization of the R^2 term, their contribution being proportional to an index, the different possibilities of embedding the spin connection into the gauge group affect the result. In some sense, the effective coupling is sensitive to the choice of gauge group, and, although the tree level contribution to this coupling is universal, in order to compute the one-loop correction we need to know explicitly the partition function.

The one-loop contribution is obtained by inserting in the vacuum amplitude the operators corresponding to (5.1). The left-moving part of any such operator is the helicity operator Q^2, that leads to the saturation of the fermion zero modes. The right moving parts are differential operators that, after the saturation of the zero modes, act on various terms of the second helicity supertrace, B_2.[2] In order to compute these quantities, we need an explicit expression of the partition function. Unfortunately, it is not possible to construct the heterotic dual at the $\mathrm{U}(1)^{16}$ point by using only discrete, $\mathbb{Z}_2$-Wilson lines. This is a technical fact, due to the strong constraints imposed, on the heterotic string, by modular invariance. However, we can bypass this difficulty by constructing the heterotic model at an extended symmetry point, the $\mathrm{U}(2)^8$ point. Even though at this point the number of hypermultiplets of the twisted sector is twice that of the type-IIB orbifold, the conclusions we will get are valid also at the $\mathrm{U}(1)^{16}$ point. As we will see, in passing from the $\mathrm{U}(2)^8$ to the $\mathrm{U}(1)^{16}$ point, both the numbers of vector and "untwisted" hypermultiplets are reduced, but their difference $N_V - N_H$ remains the same. Universality properties (see ref. [21]) then tell us that the "$\mathcal{N}_4 = 2$" sectors of the two orbifolds are the same, and this is enough for our analysis, because these are the only sectors that contribute to the helicity supertrace B_2. The partition function at the $\mathrm{U}(2)^8$ point is:

$$\begin{aligned} Z_{\mathrm{Het}} = & \frac{1}{\mathrm{Im}\,\tau|\eta|^4}\frac{1}{2}\sum_{H,G} Z_{6,22}\begin{bmatrix} H \\ G \end{bmatrix} \times \\ & \times \frac{1}{2}\sum_{a,b}\frac{1}{\eta^4}(-)^{a+b+ab}\vartheta\begin{bmatrix} a \\ b \end{bmatrix}^2 \vartheta\begin{bmatrix} a+H \\ b+G \end{bmatrix}\vartheta\begin{bmatrix} a-H \\ b-G \end{bmatrix}, \end{aligned} \tag{5.7}$$

where $\{H, G\} \in \{0, 1\}$ parametrize the boundary conditions introduced by the $\mathbb{Z}_2$ orbifold projection in the directions 1 and τ of the world-sheet torus; the second line of (5.7) stands for the contribution of the 10 left-moving world-sheet fermions ψ^μ, Ψ^I and the ghosts β, γ of the super-reparametrization; $Z_{6,22}\left[\begin{smallmatrix} H \\ G \end{smallmatrix}\right]$ accounts for the $(6, 6)$ compactified coordinates and the $c = (0, 16)$ conformal system, which is described

[2] For a definition of helicity supertraces, see for instance ref. [20], and, for a computation in the context of heterotic $\mathcal{N}_4 = 2$ orbifolds, ref. [21].

JHEP03(2001)024

by 32 right-moving fermions, Ψ_A, $A = 1, \ldots, 32$:

$$Z_{6,22}\begin{bmatrix} H \\ G \end{bmatrix} = \frac{1}{2^4}\sum_{\vec{h},\vec{g}} \frac{1}{\eta^6\bar\eta^6}\Gamma_{2,2}\begin{bmatrix} 0 \\ 0 \end{bmatrix}\,\Gamma_{4,4}\begin{bmatrix} H|\vec{h} \\ G|\vec{g} \end{bmatrix}\,\overline{\Phi}\begin{bmatrix} H,\vec{h} \\ G,\vec{g} \end{bmatrix}(-1)^{hg}\,, \tag{5.8}$$

with

$$\begin{aligned}\Phi\begin{bmatrix} H,\vec{h} \\ G,\vec{g} \end{bmatrix} = \frac{1}{2}\sum_{\gamma,\delta}\frac{1}{\eta^{16}} \times \Bigg\{&\theta\begin{bmatrix} \gamma+h_1+h \\ \delta+g_1+g \end{bmatrix}\theta\begin{bmatrix} \gamma+h_1-h \\ \delta+g_1-g \end{bmatrix}\theta^2\begin{bmatrix} \gamma+h_1 \\ \delta+g_1 \end{bmatrix}\times\\ &\times\Bigg(\theta\begin{bmatrix} \gamma+h_1+h_2 \\ \delta+g_1+g_2 \end{bmatrix}\theta\begin{bmatrix} \gamma+h_1-h_2-h_3 \\ \delta+g_1-g_2-g_3 \end{bmatrix}\times\\ &\times\,\theta\begin{bmatrix} \gamma+h_1+h_2-H \\ \delta+g_1+g_2-G \end{bmatrix}\times\\ &\times\theta\begin{bmatrix} \gamma+h_1+h_2+h_3+H \\ \delta+g_1+g_2+g_3+G \end{bmatrix}\Bigg)\Bigg\}\times\\ \times\Bigg\{&\theta\begin{bmatrix} \gamma+H+h \\ \delta+G+g \end{bmatrix}\theta\begin{bmatrix} \gamma+H-h \\ \delta+G-g \end{bmatrix}\theta^2\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\times\\ &\times\Bigg(\theta\begin{bmatrix} \gamma+h_2 \\ \delta+g_2 \end{bmatrix}\theta\begin{bmatrix} \gamma-h_2-h_3 \\ \delta-g_2-g_3 \end{bmatrix}\theta\begin{bmatrix} \gamma+h_2+H \\ \delta+g_2+G \end{bmatrix}\times\\ &\times\theta\begin{bmatrix} \gamma+h_2+h_3-H \\ \delta+g_2+g_3-G \end{bmatrix}\Bigg)\Bigg\}.\end{aligned} \tag{5.9}$$

Here, $(\vec{h},\vec{g}) \equiv \{(h,g),(h_1,g_1),(h_2,g_2),(h_3,g_3)\}$ indicates the projections introduced by the Wilson lines: Y_1, that breaks SO(32) to SO(16)×SO(16), Y, that symmetrizes the action of $Z_2^{(H,G)}$ in the two SO(16) factors, and Y_2, Y_3, that break further the gauge group to $(\mathrm{SO}(4)\times\mathrm{U}(1)\times\mathrm{U}(1))^4 \cong \mathrm{U}(2)^8$. In order to make eq. (5.9) easily readable, we have separated the two "SO(16)" blocks by braces $\{\}\times\{\}$. Within these blocks, the characters corresponding to the $\mathrm{U}(1)^4$ factors are collected in the brackets ( ). The contribution of the bosons of the compact space is written in terms of shifted-twisted lattice sums: $\Gamma_{2,2}\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$ is the contribution of the two-torus, and the arguments $\left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}\right]$ indicate that there is no shift of the momenta; $\Gamma_{4,4}\left[\begin{smallmatrix} H|\vec{h} \\ G|\vec{g} \end{smallmatrix}\right]$ is the contribution of the twisted $\mathbb{T}^4$, where the first column in the argument indicates the twist, the second indicates the shifts produced by the Wilson lines.[3] The spectrum of the untwisted sector of this orbifold contains the vector multiplets of the $\mathrm{U}(1)^2$ of the two-torus and of the adjoint representation of the gauge group $\mathrm{U}(2)^8$; the hypermultiplets are: four from the twisted $\mathbb{T}^4$ and sixteen in the $(\mathbf{2},\mathbf{2})$ of the four factors $\mathrm{SO}(2)\times\mathrm{SO}(2)\cong\mathrm{U}(1)\times\mathrm{U}(1)$. In the twisted sector, there are $(1/2)\times 16$ hypermultiplets in the $(\mathbf{2},\mathbf{1})$ and $(1/2)\times 16$ in the $(\mathbf{1},\mathbf{2})$ of spin(4)⊗spin(4). When the $\mathrm{U}(2)^8$ gauge group is broken to $\mathrm{U}(1)^{16}$, we obtain precisely sixteen hypermultiplets from the twisted sector.

[3]For more details on twisted-shifted lattice sums, we refer the reader to [13, 21].

As we said, the relevant quantity for the one-loop computation of the gravitational corrections is the second helicity supertrace, B_2. After a straightforward computation, we obtain:

$$B_2 = \frac{1}{\overline{\eta}^{24}} \sum_{(H,G)}{}' \, \Gamma_{2,2} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \overline{\Omega} \begin{bmatrix} H \\ G \end{bmatrix} , \tag{5.10}$$

where the prime indicates that the sum is taken only over the values $(H,G)= \{(0,1),$ $(1,0)$, $(1,1)\}$. In our case, the modular forms $\Omega \begin{bmatrix} H \\ G \end{bmatrix}$ read:

$$\begin{aligned} \Omega \begin{bmatrix} 0 \\ 1 \end{bmatrix} &= \frac{1}{2} \left(\vartheta_3^4 + \vartheta_4^4\right) \vartheta_3^8 \, \vartheta_4^8 \, , \\ \Omega \begin{bmatrix} 1 \\ 0 \end{bmatrix} &= -\frac{1}{2} \left(\vartheta_2^4 + \vartheta_3^4\right) \vartheta_2^8 \, \vartheta_3^8 \, , \\ \Omega \begin{bmatrix} 1 \\ 1 \end{bmatrix} &= \frac{1}{2} \left(\vartheta_2^4 - \vartheta_4^4\right) \vartheta_2^8 \, \vartheta_4^8 \, . \end{aligned} \tag{5.11}$$

Acting on B_2 with the right-moving part of (5.1), $\overline{R}^2_{(\mathrm{o})}$, we obtain:

$$\overline{R}^2_{(\mathrm{o})} B_2 = \frac{2}{3} \sum_{(H,G)}{}' \, \Gamma_{2,2} \begin{bmatrix} 0 \\ 0 \end{bmatrix} . \tag{5.12}$$

After infrared-regularization [22] and integration over the fundamental domain, we finally get the one loop contribution,[4] that, summed to that of tree level, gives:

$$\frac{16\,\pi^2}{g^2} = 16\,\pi^2 \,\mathrm{Im}\, S + \Delta^{N^{(\mathrm{t})}=16}(T,U) + \frac{b \log M_{\mathrm{P}}}{\sqrt{p^2}} \, , \tag{5.13}$$

where the function $\Delta^{N^{(\mathrm{t})}=16}(T,U)$ is given as in eq. (5.4), with T and U the moduli associated, respectively to the Kähler class and the complex structure of $\mathbb{T}^2$. In the orbifolds with a reduced number of fixed points, the dependence on the moduli T and U is different. In the case of half number of fixed points, i.e. with $N^{(\mathrm{t})} = 8$, the contribution of the two-torus, instead of $\Gamma_{2,2} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, is, both on the type-IIB and heterotic sides:

$$\frac{1}{2} \Gamma_{2,2} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \frac{1}{2} \Gamma_{2,2} \begin{bmatrix} H \\ G \end{bmatrix} , \tag{5.14}$$

where the arguments indicate the shift produced by a $\mathbb{Z}_2$ translation in one circle of the two-torus. In the case of one-quarter of fixed points, $N^{(\mathrm{t})} = 4$, it is:

$$\frac{1}{4} \Gamma_{2,2} \begin{bmatrix} 0,0 \\ 0,0 \end{bmatrix} + \frac{1}{4} \Gamma_{2,2} \begin{bmatrix} H,0 \\ G,0 \end{bmatrix} + \frac{1}{4} \Gamma_{2,2} \begin{bmatrix} 0,H \\ 0,G \end{bmatrix} + \frac{1}{4} \Gamma_{2,2} \begin{bmatrix} H,H \\ G,G \end{bmatrix} , \tag{5.15}$$

[4] Notice that $\Omega \begin{bmatrix} H \\ G \end{bmatrix}$ is twice the $\Omega^{(0)} \begin{bmatrix} H \\ G \end{bmatrix}$ part of the helicity supertrace in the $N_V = N_H$ freely acting orbifolds considered in refs. [12, 18, 23].

where now there are two independent shifts, along the two independent circles of the two-torus. In the first case, the function $\Delta^{N^{(\mathrm{t})}}(T,U) = \Delta^{N^{(\mathrm{t})}}(T) + \Delta^{N^{(\mathrm{t})}}(U)$, that encodes the (T,U)-dependence of the correction on the heterotic side, and whose U-dependent part gives the U-dependence of the type-I corrections, is:

$$\begin{aligned}\Delta^{8}\left(T,U\right) = &-\log \mathrm{Im}\, T \left|\eta\left(T\right)\right|^{4} - \frac{1}{3}\log \mathrm{Im}\, T \left|\vartheta_{4}\left(T\right)\right|^{4} - \\ &- \log \mathrm{Im}\, U \left|\eta\left(U\right)\right|^{4} - \frac{1}{3}\log \mathrm{Im}\, U \left|\vartheta_{4}\left(U\right)\right|^{4}. \end{aligned} \tag{5.16}$$

In the second case,

$$\begin{aligned}\Delta^{4}\left(T,U\right) = &-\frac{1}{2}\log \mathrm{Im}\, T \left|\eta\left(T\right)\right|^{4} - \frac{1}{2}\log \mathrm{Im}\, T \left|\vartheta_{4}\left(T\right)\right|^{4} - \\ &- \frac{1}{2}\log \mathrm{Im}\, U \left|\eta\left(U\right)\right|^{4} - \frac{1}{2}\log \mathrm{Im}\, U \left|\eta\left(U\right)\right|^{4}. \end{aligned} \tag{5.17}$$

The coefficients of the $\log \mathrm{Im}\, T$, $\log \mathrm{Im}\, U$ terms are the beta-function coefficients of the gravitational term as seen on the heterotic side, at the abelian point:

$$b_{\mathrm{grav}} = \frac{24 + N^{(\mathrm{t})} - (N_V - N_H)}{12}\,, \tag{5.18}$$

where $(N_V - N_H)$, the contribution of the fields originating from the currents, is 16. We stress that $(N_V - N_H) = 16$ at both the $\mathrm{U}(2)^8$ and $\mathrm{U}(1)^{16}$ points. As a consequence, also the functions $\Omega \begin{bmatrix} H \\ G \end{bmatrix}$ are the same. As we mentioned, this is due to an universality property, analyzed in ref. ([21]) for a class of heterotic models in which the two-torus lattice sum is $\begin{bmatrix} H \\ G \end{bmatrix}$-shifted. The universality properties there derived, relating models with the same shift in the lattice sum and the same $N_V - N_H$, are valid here, if N_V, N_H are the numbers of vector and hypermultiplets in the orbifold untwisted sector.

In the $N^{(\mathrm{t})} = 16$ case, without the $\mathcal{D}$-projections, the result (5.12) could have been obtained in a more economic way, without knowing the details of the partition function in the $\begin{bmatrix} H \\ G \end{bmatrix}$-twisted sector, by using general properties of the amplitudes. In that case, in fact, it is possible to factorize the contribution of the two-torus lattice sum, $\Gamma_{2,2} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, the only piece that contains a dependence on the moduli, and sum over (H,G) to obtain directly the amplitude $\langle R^2_{(\mathrm{o})} \rangle$. Owing to the cancellation, in the integrand, of the non-holomorphic pieces proportional to $1/\mathrm{Im}\,\tau$, the latter has the general form [24]:

$$\left\langle R^2_{(\mathrm{o})} \right\rangle = \int_{\mathcal{F}} \frac{d^2\tau}{\mathrm{Im}\,\tau} \Gamma_{2,2} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \left(A\bar{j}(\bar{\tau}) + B\right). \tag{5.19}$$

A and B are constants, and it is intended that the integral has to be properly regularized by subtracting the infrared divergence. In order to conclude for the result, it is then enough to observe that the singularity for $\tau \to \mathrm{i}\infty$ in the integrand of (5.19), due to the pole of the j-function, is subtracted by $\langle P^2 \rangle_{(\mathbb{T}^2)}$ and that the subtraction of this amplitude, which has a vanishing beta-function coefficient, does not change the constant term B, that remains equal to the beta-function coefficient of $\langle R^2 \rangle$. This implies that $A = 0$ and $B = b_{\mathrm{grav}}$.

Comparison of the three constructions. We pass now to the comparison of the $\tilde{\Omega}$ constructions with the heterotic and type-I models. Since the $\tilde{\Omega}$ orbifolds match these two string constructions in a corner of their moduli space, they can be considered as particular deformations of the heterotic/type-I string. We start therefore with the analysis in the region of the moduli space in which these theories match.

The heterotic/type-I duality for $\mathbb{Z}_2$, non-freely acting orbifolds was analyzed in ref. [8, 9], where it was found that the T-dependent, one-loop heterotic contribution to the effective coupling is mapped, on the type-I side, into a tree-level plus a non-perturbative contribution. In the large-T limit, $\Delta^{N^{(\mathrm{t})}}(T)$ behaves as $\sim \frac{\pi}{24} N^{(\mathrm{t})} \, \mathrm{Im}\, T$, and the heterotic one-loop contribution reproduces the term in eq. (5.2) linear in S', that parametrizes the coupling of the gauge fields of the D5-branes sector. The map is therefore $N^{(\mathrm{t})}\, T \to 3 \times 2^7 \, \pi \, S'$. The coefficient $N^{(\mathrm{t})}$ automatically takes into account the fact that, when the number of twisted hypermultiplets is reduced, there is a "higher level" realization of the D5-branes gauge group, reflected in a different strength of the coupling: $\mathrm{Im}\, S'_{2k} = 2 \, \mathrm{Im}\, S'_k$, where the level k is related to the rank of the gauge group by $k = 16/r$. As usual, the heterotic dilaton-axion field S is dual to the field S of type I. In the case of the dual type-I orientifolds obtained by T-dualizing the four coordinates transverse to the D5-branes, the map is similar, with a simple exchange of the role of the fields S and S'. This is also the expression of the effective coupling in the $\tilde{\Omega}$ constructions, for $V_{(4)} \to 0$. In the opposite limit, $V_{(4)} \to \infty$, the correction is on the other hand approximated by the expression given in eqs. (5.5). Matching these constructions with the heterotic theory leads to conclude that indeed the dominant behavior of the contribution of the $\tilde{\Omega}$ D9-branes sector to the effective coupling is given by (5.4), with the following identifications:

$$T \longleftrightarrow S' \longleftrightarrow \tilde{S}^{-1} \,. \tag{5.20}$$

At the point in which these theories match with the heterotic string, we can trade the symmetries in the heterotic field T for symmetries of the corresponding dual field $\tilde{S}$ in the $\tilde{\Omega}$ constructions. Since these can be seen as deformations "toward the bulk" of type-I orientifolds, we argue that such symmetries continue to hold also away from the matching point. For instance, in the orbifold with sixteen fixed points, in order to understand what happens when we invert the field $\tilde{S}$, we can consider to go first to the point at which the theory matches the heterotic string, and invert there the field $S' \propto \tilde{S}^{-1}$. On the heterotic side, this corresponds to an inversion of the field T. Since we are at the point at which the two theories match, the invariance of the heterotic theory under inversion of T must reflect in an analogous invariance of the $\tilde{\Omega}$ theory under inversion of $\tilde{S}$. We can now deform back this theory to a finite $V_{(4)}$: the new theory must coincide with the one before inversion of $\tilde{S}$.

In the $V_{(4)} \to \infty$ limit, the contribution of the field S, corresponding to the tree level coupling of the D5-branes of the $\tilde{\Omega}$ orbifolds, is on the other hand missing. As it was for the cases considered in ref. [1], also in this case, from the point of view of the effective theory felt by an observer on a four-dimensional wall placed at some point in the bulk, we interpret this decompactification as a non-perturbative motion in the field S. The absence of a contribution linear in this field suggests that the linear term, corresponding to a tree level contribution on the heterotic side, may come from the expansion of a function of the type:

$$\sim \log \mathrm{Im}\, S |\vartheta_2(S)|^4 \, + \, \mathcal{O}\left(\mathrm{e}^{-S} f(T,U)\right), \tag{5.21}$$

where the second term is a series of terms exponentially suppressed both in the large- and small-S limits. For large $\mathrm{Im}\, S$, expression (5.21) reproduces the tree level linear dependence on this field, while for $S \to 0$, this function does not vanish but diverges only logarithmically in $-1/S$. The theory behaves therefore as if we were in the presence of a non-perturbative Scherk-Schwarz mechanism, providing a phenomenon analogous to what happens, perturbatively, in the semi-freely acting orbifolds.[5] We argue that the total correction to the effective coupling of the R^2 term takes the following form:

$$\frac{16\,\pi^2}{g^2} \approx 16\,\pi \log \mathrm{Im}\, S |\vartheta_2(S)|^4 + \Delta^{N^{(\mathrm{t})}}(\tilde{S}^{-1}, U) + \frac{b \log M_{\mathrm{P}}}{\sqrt{p^2}} + \\ + \mathcal{O}\left(\mathrm{e}^{-S} f(\tilde{S}^{-1}, U)\right). \tag{5.22}$$

In the above expression, we must allow for the presence of terms, suppressed in both the large- and small-S limits, that mix the contributions of the three moduli, S, $\tilde{S}$, U.

The gauge couplings. The analysis of the effective coupling of the R^2 term allows us to learn something also about the effective couplings of the gauge group in the $\tilde{\Omega}$ constructions. From the bulk point of view, the states living on the D5-branes appear to be massive. This is true not only at the abelian point, but also for configurations with a larger gauge group, such as those one can obtain by considering Dp-branes, $p > 5$, via T-dualization of some coordinates and switching-off of some Wilson lines. These are the states corresponding to the heterotic perturbative gauge group. The $\tilde{\Omega}$ constructions represent therefore non-perturbative deformations of the heterotic string in which, owing to a non-perturbative Higgs phenomenon, all the states of

[5] See for instance ref. [14].

the "currents" acquire a mass, depending on the field S. The states of the D9-branes of the $\tilde{\Omega}$ constructions, on the other hand, are generically massless (we are not considering here possible mass terms required by anomaly cancellation at the contact point with the heterotic/type-I theory). The only massless gauge bosons of these constructions originate from this sector, entirely non perturbative from the heterotic point of view, with coupling parametrized by the field S'; the analysis of the previous section tells us also that the effective coupling is essentially invariant under inversion of this field. We expect therefore, for the orbifold with maximal number of fixed points, a behavior roughly of the type:

$$\frac{1}{\alpha} \approx \log \mathrm{Im}\, \tilde{S} |\eta(\tilde{S})|^4 \,. \tag{5.23}$$

(In the models with a reduced rank of the gauge group, the approximate behavior is obtained instead from the expressions (5.16) and (5.17)). At a generic point in the moduli space of this theory, however, the gauge beta function is not vanishing, and this expression may receive corrections that depend, among the others, also on the field S, that parametrizes the couplings of the D5-branes sector; this implies that this sector "feels" the presence of the other one. Expression (5.23) then simply gives the dominant behavior of the coupling, as a function of one modulus, indicating the existence of an (approximate?) S-duality. The coupling of the (massive) states of the other sector behaves on the other hand as:

$$\frac{1}{\tilde{\alpha}} \approx \log \mathrm{Im}\, S |\vartheta(S)|^4 \,, \tag{5.24}$$

and the mass of these states scales as the inverse of the field S. In the limit $S \to \infty$, corresponding to the heterotic weak coupling, these states become massless, and the gauge group gets enhanced.

6. Conclusions

In this work we have analyzed the bulk theory of a class of $\mathbb{Z}_2$ orbifolds of the type-IIB string, compactified to four space-time dimensions, in the presence of D5-branes. Cancellation of the RR charge fixes uniquely the number of D5-branes. The $\mathbb{Z}_2$ orbifold projection, parallel to the D5-branes, adds to the spectrum the states of the orbifold fixed points, that appear as D9-branes. From an observer sitting on a four-dimensional probe located at a point in the bulk, supersymmetry appears broken to $\mathcal{N}_4 = 2$ even outside of the D5-branes. This is due to the interaction of branes and bulk states. In the limit of compactification to zero size of the space transverse to the D5-branes, the effective theory is expected to match with that of an orbifold of the type-I string (and, by duality, also heterotic string), compactified to four dimensions. On the other hand, when the transverse space is decompactified, we expect that, in the case in which there is a local cancellation of the RR charge between D5 branes and corresponding orientifold planes, from the bulk point of view the D5-branes effects appear to be suppressed. This however does not necessarily lead to a restoration of a larger amount of supersymmetry in the bulk: when the orbifold projection has fixed points, there is indeed no point in the moduli space of these constructions in which there is such restoration. We have considered the effective four-dimensional action and the corrections to the coupling of the R^2 term. By matching them, at a corner in their moduli space, with heterotic orbifolds, we interpreted these constructions as providing a non-perturbative deformations of the heterotic/type-I string. The motion toward a decompactification regime can then be seen, from the point of view of the heterotic four-dimensional effective theory, as a non-perturbative motion toward a particular strong coupling regime. This interpretation is somehow reminiscent of the interpretation of the decompactification of the eleventh coordinate of the M-theory as a motion toward a strong string coupling regime. Indeed, we believe that these constructions, as well as those considered in ref. [1], provide effective realizations of Scherk-Schwarz mechanisms that lift the mass of the heterotic perturbative currents; they probably have their appropriate realization as (semi)freely acting orbifolds of higher-dimensional theories, a hint of whom can be found in refs. [27]–[29]. In this framework, the constructions in which the orbifold projection acts freely would then realize phases in which all the gauge bosons are massive, while those in which the projections acts only semi-freely have massless gauge bosons associated to the orbifold fixed points. Such bosons can be entirely non perturbative from the heterotic point of view: this is indeed the case considered in this work, and is realized when they are associated to the fixed points of a projection non perturbative from the point of view of the heterotic string.

Acknowledgments

I thank C. Angelantonj, I. Antoniadis, C. Bachas, K. Benakli, R. Blumenhagen, D. Lüst, P. Mayr, W. Nahm, Y. Oz and A. Sagnotti for valuable discussions, the École Normale of Paris and the CERN Theory Division for hospitality.

References

[1] A. Gregori, *Type-IIB orbifolds with D5-branes and their string duals*, *Nucl. Phys.* **B 588** (2000) 178 [hep-th/0004070].

[2] R. Blumenhagen, C. Kounnas and D. Lust, *J. High Energy Phys.* **01** (2000) 036.

[3] See the discussion in: J. Polchinski and E. Witten, *Evidence for heterotic–type-I string duality*, *Nucl. Phys.* **B 460** (1996) 525 [hep-th/9510169].

[4] M. Bianchi and A. Sagnotti, *Twist symmetry and open string Wilson lines*, *Nucl. Phys.* **B 361** (1991) 519.

[5] E.G. Gimon and J. Polchinski, *Consistency conditions for orientifolds and D-manifolds*, *Phys. Rev.* **D 54** (1996) 1667 [hep-th/9601038].

[6] M. Berkooz, R. G. Leigh, J. Polchinski, J. H. Schwarz, N. Seiberg and Edward Witten, *Anomalies, dualities and topology of $D = 6$ $N = 1$ superstring vacua*, *Nucl. Phys.* **B 475** (1996) 115 [hep-th/9605184].

[7] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, *Phys. Lett.* **B 294** (1992) 196 [hep-th/9210127].

[8] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T.R. Taylor, *Aspects of type-I–type-II–heterotic triality in four dimensions*, *Nucl. Phys.* **B 489** (1997) 160 [hep-th/9608012].

[9] I. Antoniadis, H. Partouche and T.R. Taylor, *Duality of $N = 2$ heterotic–type-I compactifications in four dimensions*, *Nucl. Phys.* **B 499** (1997) 29 [hep-th/9703076];
I. Antoniadis, H. Partouche and T.R. Taylor, *Lectures on heterotic type-I duality*, *Nucl. Phys.* **61A** *(Proc. Suppl.)* (1998) 58 [hep-th/9706211].

[10] M.J. Duff, R. Minasian and E. Witten, *Evidence for heterotic/heterotic duality*, *Nucl. Phys.* **B 465** (1996) 413 [hep-th/9601036].

[11] R. Dijkgraaf, E. Verlinde and H. Verlinde, *$C = 1$ conformal field theories on Riemann surfaces*, *Comm. Math. Phys.* **115** (1988) 649.

[12] A. Gregori, C. Kounnas and P.M. Petropoulos, *Non-perturbative gravitational corrections in a class of $N = 2$ string duals*, *Nucl. Phys.* **B 537** (1999) 317 [hep-th/9808024];
A. Gregori, *Nonperturbative corrections in $N = 2$ strings*, talk given at the *6th Hellenic School and Workshop on Elementary Particle Physics*, Corfu, Greece, 6–26 Sep 1998, in *Corfu 1998, Quantum aspects of gauge theories, supersymmetry and unification* pp. 348–355 [hep-th/9811096].

[13] A. Gregori, E. Kiritsis, C. Kounnas, N.A. Obers, P.M. Petropoulos and B. Pioline, *R^2 corrections and non-perturbative dualities of $N = 4$ string ground states*, *Nucl. Phys.* **B 510** (1998) 423 [hep-th/9708062].

[14] A. Gregori, C. Kounnas and J. Rizos, *Classification of the $N = 2$, $\mathbb{Z}_2 \times \mathbb{Z}_2$-symmetric type-II orbifolds and their type-II asymmetric duals*, *Nucl. Phys.* **B 549** (1999) 16 [hep-th/9901123].

[15] E. Witten, *Small instantons in string theory*, *Nucl. Phys.* **B 460** (1996) 541 [hep-th/9511030].

[16] I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti, *Partial breaking of supersymmetry, open strings and M-theory*, *Nucl. Phys.* **B 553** (1999) 133 [hep-th/9812118].

[17] G. Aldazabal, A. Font, L.E. Ibáñez and F. Quevedo, *Heterotic/heterotic duality in $D = 6, 4$*, *Phys. Lett.* **B 380** (1996) 33 [hep-th/9602097].

[18] A. Gregori and C. Kounnas, *Four-dimensional $N = 2$ superstring constructions and their (non-)perturbative duality connections*, *Nucl. Phys.* **B 560** (1999) 135 [hep-th/9904151];
A. Gregori, *Non-perturbative string connections*, in the proceedings of the 9'th TMR meeting on *Quantum aspects of gauge theories, supersymmetry and unification*, Paris, France, 1–7 September 1999 [hep-th/0002214].

[19] M. Serone, *$N = 2$ type-I heterotic duality and higher derivative F-terms*, *Phys. Lett.* **B 395** (1997) 42 [hep-th/9611017], erratum *ibid.* **B401** (1997) 363;
J.F. Morales and M. Serone, *Higher derivative f-terms in $N = 2$ strings*, *Nucl. Phys.* **B 481** (1996) 389 [hep-th/9607193].

[20] E. Kiritsis, *Introduction to superstring theory*, lectures presented at the Catholic University of Leuven and University of Padova, 1996-1997, Leuven, Belgium, Leuven Univ. Pr. 1998, *Leuven notes in mathematical and theoretical physics*, **B9**[hep-th/9709062].

[21] E. Kiritsis, C. Kounnas, P.M. Petropoulos and J. Rizos, *String threshold corrections in models with spontaneously broken supersymmetry*, *Nucl. Phys.* **B 540** (1999) 87 [hep-th/9807067].

[22] E. Kiritsis and C. Kounnas, *Infrared regularization of superstring theory and the one loop calculation of coupling constants*, *Nucl. Phys.* **B 442** (1995) 472 [hep-th/9501020].

[23] A. Gregori, C. Kounnas and P.M. Petropoulos, *Non-perturbative triality in heterotic and type-II $N = 2$ strings*, *Nucl. Phys.* **B 553** (1999) 108 [hep-th/9901117].

[24] E. Kiritsis, C. Kounnas, P.M. Petropoulos and J. Rizos, *Universality properties of $N = 2$ and $N = 1$ heterotic threshold corrections*, *Nucl. Phys.* **B 483** (1997) 141 [hep-th/9608034]; *On the heterotic effective action at one-loop, gauge couplings and the gravitational sector*, in the proceedings of *5th Hellenic School and Workshops on Elementary Particle Physics*, Corfu, Greece, 3-24 Sep 1995 [hep-th/9605011].

[25] L.J. Dixon, V. Kaplunovsky and J. Louis, *Moduli dependence of string loop corrections to gauge coupling constants*, *Nucl. Phys.* **B 355** (1991) 649;
P.M. Petropoulos and J. Rizos, *Universal moduli-dependent thresholds in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds*, *Phys. Lett.* **B 374** (1996) 49 [hep-th/9601037].

[26] C.P. Bachas, M.B. Green and A. Schwimmer, *(8,0) quantum mechanics and symmetry enhancement in type-I' superstrings*, *J. High Energy Phys.* **01** (1998) 006 [hep-th/9712086].

[27] C. Kounnas, *Conformal string sector of M-theory*, alk given at TMR Meeting on *Quantum aspects of gauge theories, supersymmetry and unification*, Paris, France, 1-7 Sep 1999, hep-th/0005271.

[28] I. Bars and C. Kounnas, *Theories with two times*, *Phys. Lett.* **B 402** (1997) 25 [hep-th/9703060]; *String and particle with two times*, *Phys. Rev.* **D 56** (1997) 3664 [hep-th/9705205].

[29] I. Bars, *A case for 14 dimensions*, *Phys. Lett.* **B 403** (1997) 257 [hep-th/9704054].